Cashbacks falling from the sky: Can retail CBDC rollout widen central banks' toolkit?*

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Abstract

This paper proposes a new digital-currency-enabled monetary policy tool: sector-specific cashback payments on private consumption expenditure. Using a two-sector New Keynesian model, we show how the monetary policy toolkit can benefit from adopting such instruments in two important instances. First, cashback payments provide a way to stimulate aggregate demand when the policy rate is at the zero lower bound without relying on the intermediate role of financial or corporate sectors in propagating liquidity injections. Secondly, we demonstrate how the adoption of simple cashback rate rules can produce a more efficient allocation of resources under asymmetric shocks compared to a simple implementable interest rate rule alone. The analysis proceeds to draw relationships between the size of the welfare gain associated with the introduction of these novel instruments and degrees of price stickiness, labour mobility, and relative size of the sectors.

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I Introduction

While private initiatives such as Bitcoin(s), Ethereum, and Libra have led to the introduction of digital currencies, researchers and policymakers have investigated the possibility for central banks to issue digital currency of their own. It is referred to as retail central bank digital currency (CBDC) and assumed to be available to the general public. In contrast to private initiatives, retail CBDC involves several challenges for the central banks to overcome. For instance, potential disintermediation of the banking sector, reputational and other risks, legal considerations, technical and organisational obstacles.

Central banks face older challenges still. First, the diverging inflation rates in different sectors, regions, or even countries imply welfare losses (Benigno, 2004). Be it the long-run divergence of the durable sector inflation rate from the Bank of England target in the United Kingdom or the diverging pressures on prices in the demand-constrained and supply-constrained sectors during the COVID-19 pandemic in the United States, the current status quo of the monetary policy is only partially suited to withstand the welfare-diminishing effects of asymmetric shocks. Alternatively, currency unions, with the Euro Area being a formidable example, assign country weights in inflation targeting based on the economic contributions of its members, thus, often discriminating against smaller and less developed states for the ”greater good of stability on average”. These are but a few examples of the inefficiencies in the current design of monetary policy that we believe can be partially alleviated using new, systematic, sector, region- or country-specific monetary policy instruments.

Secondly, the zero lower bound (ZLB) on interest rates hindered the ability of conventional monetary policy to ease money market conditions in some advanced economies during the Global financial crisis of 2008 and in the majority of them during the outbreak of the COVID-19 pandemic in 2020. Central banks of major advanced economies increasingly rely on wider asset-purchasing programmes to directly inject liquidity into the banking and corporate sectors, or on helicopter money for direct transfers to households' budgets. Such instruments often face frictions that diminish their desired impact on aggregate demand; banks may hesitate to lend more and households may choose to save instead of spending. Moreover, these policies are usually introduced as a tool of last resort, rather than a systematic monetary policy rule designed to achieve a certain long-term target.

The challenges posed by the adoption of retail CBDC, however, also introduce new opportunities for the central banks to deal with these old problems. Namely, the tool proposed in this paper relies on wide-spread adoption of retail CBDC and the resulting shift in the policy potential that can be achieved by the adoption of recent technological advancements. Introduction of cashback payments on private consumption expenditure, paid in retail CBDC and financed by the expansion of the central bank's balance sheet, can help the monetary authority achieve not only short-term but also long-term goals. On one hand, such instruments would allow to deliver a temporary stimulus to the economy during periods of ZLB. A stimulus similar to the helicopter money in its direct transfer nature, but, unlike it, one that cannot be set aside or saved since it is linked to consumption expenditure. On the other hand, carefully designed sectoral cashback rate rules can minimise welfare-diminishing asymmetries in sectoral inflation rates in the long-term. Overall, the potential benefits include an increase in policy space, reduction of incidences of the ZLB and a lower probability of falling into a deflationary spiral.

This paper contributes to the literature in two ways. First, we discuss how cashback rates enabled by retail CBDC are capable of complementing the modern monetary policy toolkit in two respects: by adding a policy instrument that can be used when interest rates are at the ZLB, and by allowing a monetary authority to diminish severeness of the trade-off between minimising the terms of the trade gap and stabilising sectoral inflation rates in a multi-sector models. Secondly, we produce the optimal combination of cashback and interest rate rules that deliver a more efficient allocation of resources, and hence a higher population welfare, compared to the current status quo.

The paper proceeds as follows. Section II provides a brief overview of the literature. Section III describes the proposed monetary policy instrument. Section IV describes the New Keynesian model used for the analysis. Section V evaluates performance of cashback instruments when the central bank's policy rate is at the zero lower bound. Section VI contains the welfare analysis and presents the optimised simple policy rules. Conclusion and appendices follow.

II Literature review

This paper is related to the branch of literature which examines the optimal monetary policy in the economy characterised by multiple sectors, asymmetric shocks and stickiness in price adjustments. It is a widely accepted result that the optimal monetary policy, in the presence of such nominal price rigidities, should target some weighted measure of inflation, where optimal weights are chosen on a case-by-case basis. Evidence in Aoki (2001) suggests that a two-sector economy, with one flexible and one sticky-price sector, should define the core inflation measure (to be targeted by a central bank) as the inflation rate of the sticky-price sector. Similarly, Benigno (2004) extends this analysis to a more general case and prescribes that monetary policy should stabilise a weighted average of the sectoral inflation rates if both sectors are sticky, with higher weight to be given to the sector with a higher degree of nominal rigidity. If the two sectors (regions) share the same degree of nominal rigidity, the optimal outcome is obtained by targeting a weighted average of the sectoral inflation rates, where weights coincide with the economic sizes of the sectors. Several other studies consider how interest rate rules should be designed in more complex two-sector cases that assume differences in durability of the goods produced in different sectors (Erceg and Levin, 2006; Cantelmo and Melina, 2017) and input-output interactions (Huang and Zheng, 2005; Petrella and Santoro, 2011; Petrella et al., 2017). There are also several extensions of new Keynesian models, where the headline CPI is suggested to be the target. For instance, in the presence of incomplete financial markets some authors suggest targeting headline CPI (Anand and Prasad, 2010; Anand et al., 2015).

The major problem arises when both sectors have nominal rigidities, and an efficient outcome is not feasible with a single monetary tool, as outlined in Benigno (2004). Even if monetary policy was able to stabilise both sector-specific inflation rates, it would also stabilise the terms of trade between two sectors, thus, constraining the optimal reallocation of resources following asymmetric disturbances. The terms of trade play a crucial role in balancing the burden of exerting output across sectors. Whenever there are asymmetric shocks that induce the households in one sector to work more, changes in terms of trade optimally shift part of the burden to the households in the other sector.

The second strand of literature that this paper is related to is concerned with zero lower bound. Conventional monetary policy appears incapable of delivering an adequate incentive to the economy and address recession and deflation once the zero lower bound for interest rates has been reached (Svensson, 2003). The problem is that the ZLB economy is satiated with liquidity so that the private sector is indifferent between either holding bonds or liquid assets. In such circumstances, the standard monetary policy of conducting open-market operations where the central bank increases the monetary base by buying government bonds leads to overcrowding of the private sector in the government bond market and pushing it into holding more money. However, such a policy does not lead to changes in prices or quantities in the economy. In other words, in the ZLB, expanding liquidity (the monetary base) beyond the satiation point does not affect the real economy.

If a combination of a liquidity trap and deflation causes the real interest rate to remain too high, the economy may fall further into a protracted recession and deflation. Benhabib et al. (2001) argue that the inability of a Taylor type rule to prevent the economy from falling into a deflationary spiral is a critical flaw of the Taylor rule as a guide to policy. Prolonged deflation can have severe negative effects. The rising real value of nominal debt may trigger bankruptcies for indebted firms and households and a fall in asset prices. It could be followed by deterioration by commercial banks' balance sheets when the value of their collateral falls and loans turn bad, causing a threat of financial instability.

There are a few proposed solutions to avoiding deflationary scenarios in the presence of ZLB. Firstly, the announcement of a positive inflation target or a price level targeting. Eggertsson and Woodford (2003) highlight the crucial role of the central bank's management of the inflation expectations of firms and households. This solution, however, is prone to scepticism about the practical effectiveness of the expectations channel, because inflation is regarded to be relatively sticky in the short run. Secondly, expanding the monetary base, usually coupled with purchases of long-term bonds, consequently reduces the long-term interest rates. An example of such an approach is the unconventional monetary policies that took place during the global financial crisis. Thirdly, a tax on money, this solution has caveats in practical implementation (argued by Gesell (1929) and Keynes (1936), and more recently by Buiter and Panigirtzoglou (2003) and Goodfriend (2000)). Finally, as argued by Feldstein (2002), a deflationary scenario can be avoided through the means of fiscal policy stimulus. Eggertsson and Woodford (2003) agree with Feldstein that there is a strong good case for state-contingent fiscal policy to deal with a liquidity trap, even if fiscal policy is not a very useful tool for stabilisation policy more generally.

III Cashback rates as a complement to the conventional monetary policy toolkit

On October 6th, 1979, the Federal Reserve System, operating under the chairmanship of Paul Volcker, took drastic action to counter severe inflationary pressures that were distorting the U.S. economy throughout the 1960s and 1970s. A strong and unprecedented commitment to diminish the rate of inflation laid the foundation for modern central banking and contributed substantially to the less hectic economic environment we enjoy today. At the time, economists reached a consensus regarding the importance of price stability in securing smooth economic development. Stable, and well-anchored, inflation reduces uncertainty regarding the future path of prices and wages, boosting the efficiency of both business and household intertemporal decision-making (Woodford, 2004).

Although during the following years, the goal of price stability never left the pedestal of monetary policy-making, its toolkit was meticulously improved over time. By the early 1990s, some of the central banks in advanced economies started following explicit inflation targets, i.e. adopted inflation targeting policies. Under such a framework, a central bank delivers price stability by setting a nominal anchor – a target aggregate rate of inflation. If the public trusts the central bank's commitment to meet its inflation target in the long run, such a nominal anchor can tie down expectations of future inflation to a narrow corridor surrounding this target rate, i.e. minimise the variance of the inflation rate.

The modern inflation targeting policies in advanced economies have several shortcomings. First, the relevant literature often considers the role of inflation targeting (specifically of low and stable inflation targets) on the probability of hitting the zero lower bound (Andrade et al., 2018; Mertens and Williams, 2019). Inflation targets fixed at low levels transmit into low steady-state nominal interest rates, diminishing the available space for the monetary policy to react to short-term output fluctuations.

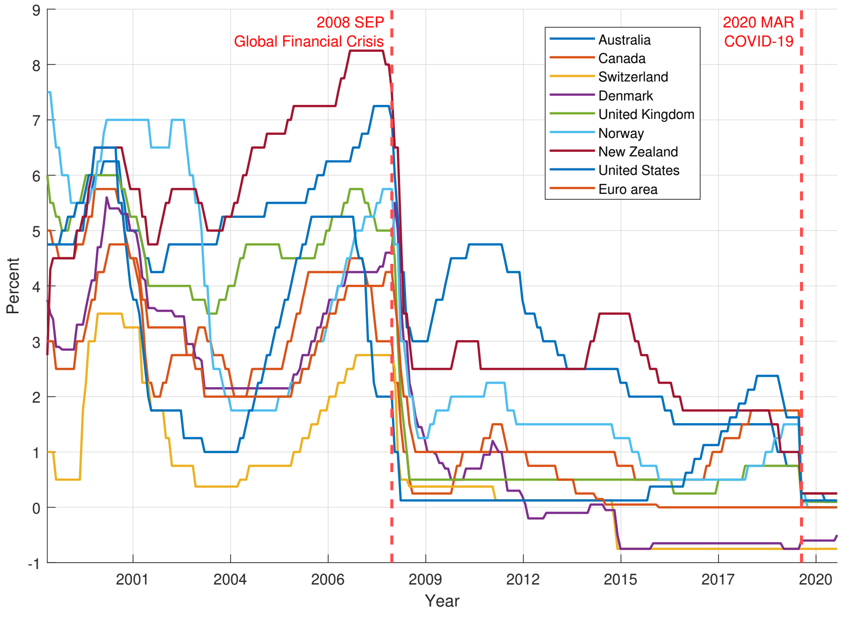

Figure 1: Policy rates of some inflation-targeting central banks.
[1]Source: Bank for International Settlements (BIS) statistics.

By way of example, to counter the adverse effects of the Global Financial Crisis, many central banks in advanced economies pushed their nominal interest rates closer to zero in 2008; policy rates in the U.K., U.S., Switzerland, and the E.U. fell into liquidity traps. Despite these efforts, such cuts in the policy rates were not sufficient to bring economies of the respective countries back on track; central banks desperately needed to bring policy rates further down but could not. As can be seen in Figure 1, many countries that managed to avoid a binding ZLB in the autumn of 2008, fell into it during the global response to the spread of COVID-19.

Another significant shortfall of the conventional inflation targeting design is that the central bank has only one systemic policy instrument - nominal short-term interest rate. As mentioned in Benigno (2004), this lack of instruments creates an environment in which inflation targeting using the harmonised index of consumer prices (HICP[2]) is optimal. Nonetheless, such a policy formulation is inferior to a hypothetical case in which a monetary authority has enough instruments to target sectoral inflation rates directly.

Most of the central banks of the advanced economies that follow inflation targeting policies are rather successful in stabilising the HICP inflation rate around the target, but not necessarily successful in stabilising the underlying sectoral inflation rates. As can be seen in Figure 2, the Bank of England was quite successful in keeping the aggregate inflation rate close to its target most of the time, an observation that cannot be extended to the inflation rate of the durable goods sector that deviated significantly from the target for prolonged periods of time.

A similar picture can be observed in the Euro Area (EA), where the European central bank targets HICP inflation composed of country-specific inflation rates, as opposed to sectoral rates discussed above.[4] Targeting a union-wide weighted average inflation rate creates heterogeneity in inflation rates and their variances. Figures 3 and 4 demonstrate this point by plotting the evolution of inflation rates across the EA between 2019 and 2020, i.e. between pre-COVID-19 and the first year of the COVID-19 pandemic. As these figures suggest, not only inflation rates differ across the EA countries within a given year, but their response to the pandemic shocks was also different.[5] A central bank targeting HICP measure attempts to minimise its variance, but will inevitably face trade-offs in minimising individual inflation variances of the

[2] HICP is a weighted average of prices in different sectors of an economy, measured using dis-aggregated data for prices

[4] From the monetary policy perspective a currency union with a common central bank and monetary policy mandate, such as the Euro Area, is only marginally different from a single country case.

[5] Whether the difference in response is driven by fixed effects or by asymmetries in the shocks is not important at this point.

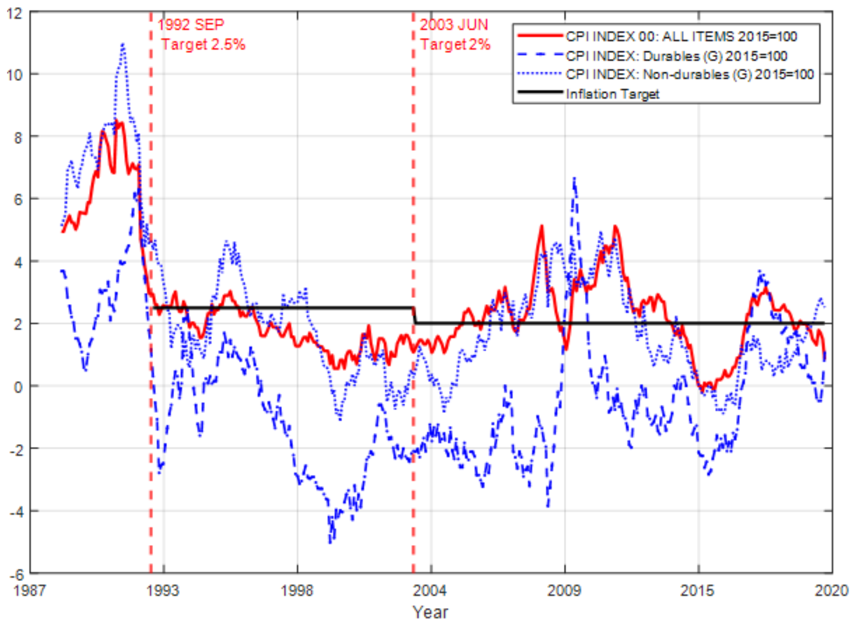

Figure 2: UK consumer price index (CPI) inflation: overall (HICP), durables sector, non-durables sector.

[3]Source: Federal Reserve Economic Database (FRED).

member countries. As argued by Benigno (2004), inability of a central bank to simultaneously achieve zero country-specific inflation variances will result in a distorted equilibrium that does not deliver a Pareto optimal outcome.

Until recently, it was hard to imagine the possibility of a central bank directly targeting inflation rates in individual sectors of an economy, or individual countries in a currency union. Indeed, the main instrument in the conventional monetary policy toolkit – the policy rate – delivers an effect on the overall economy, by influencing interest rates charged and paid by the banking sector. Thus, if a central bank wishes to increase inflation, it will decrease the policy rate, make borrowing cheaper, and, in turn, incentivise market agents to consume and invest more. Such a rise in aggregate consumption and investment expenditure transmits into higher prices as the aggregate demand for goods and services increases. A policy intervention using such an instrument cannot deliver a sector-specific effect by design.

In this regard, the concept of retail CBDC dramatically expands central banker's toolkit. One of the proposed ways to design a retail CBDC system is via a token-based solution constructed on an open blockchain. Blockchain technology is especially suitable as it provides a way to collect large quantities of transaction-specific information while effectively maintaining the pseudo-anonymity of the final user. A retail CBDC design based on a blockchain ledger can record data on the transaction amount, commodity type, and physical address of the retailer

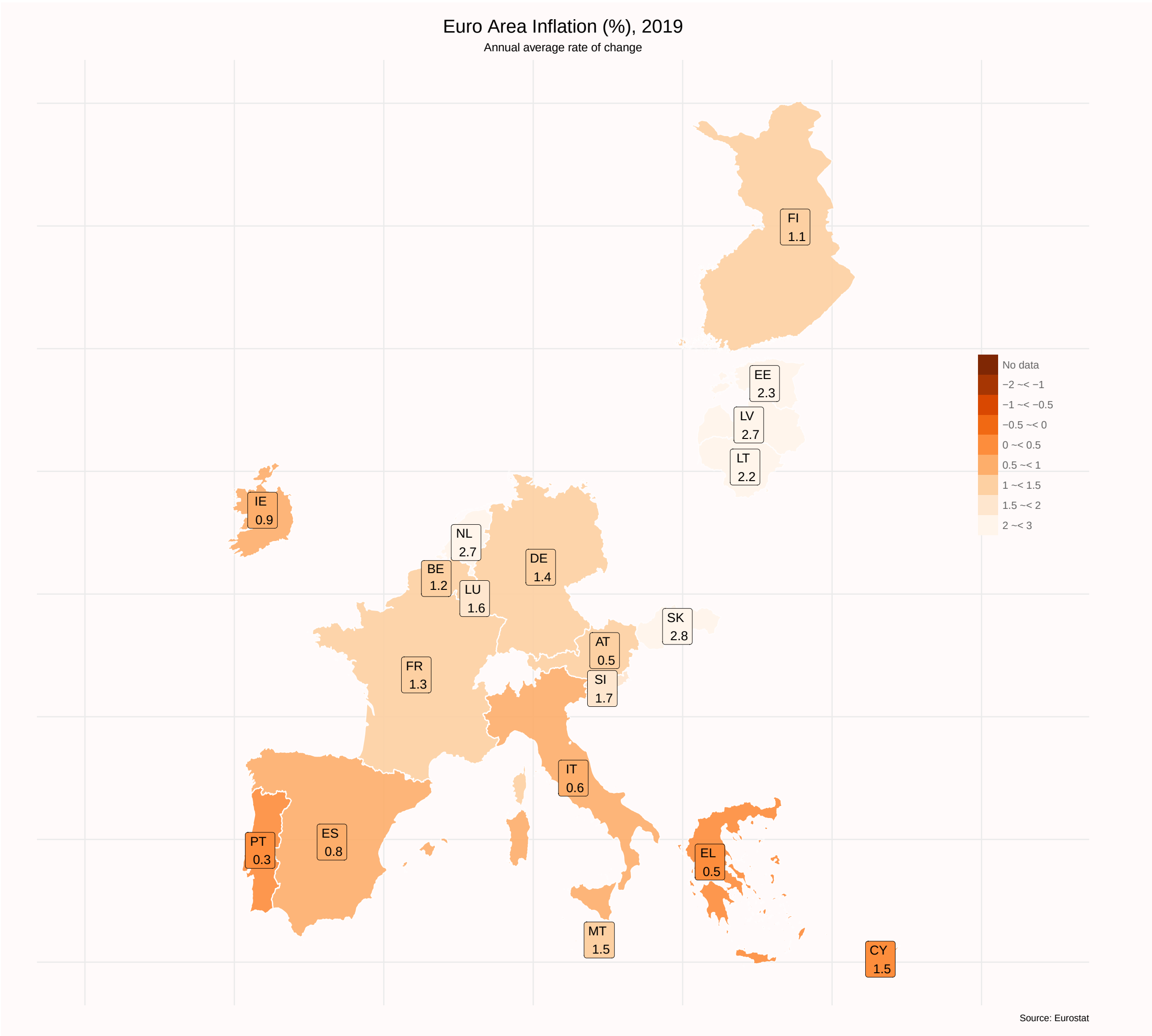

Figure 3: Annual CPI inflation rates for Euro Area member countries in 2019

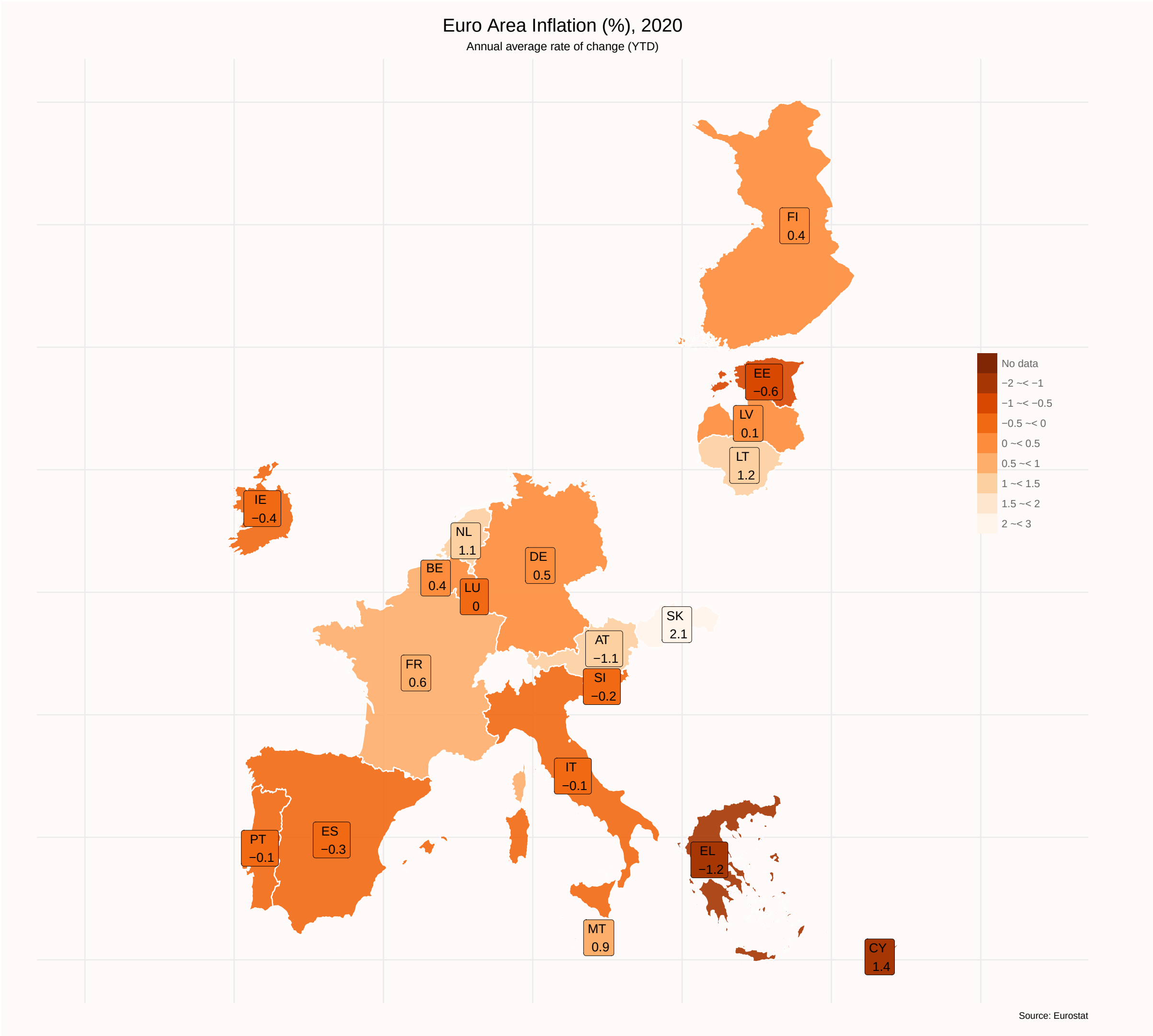

Figure 4: Annual CPI inflation rates for Euro Area member countries in 2020

or producer, without compromising sensitive personal information. In the presence of such real-time Big Data on transactions, a monetary authority can improve both the precision of economic surveillance and develop a toolkit to surgically intervene only in the segments of the economy where such interventions are most needed.[6]

We propose to introduce a system of two-way monetary transfers between a country's central bank and households. These transfers are a function of two inputs. The first input is the nominal value of household's purchases in a given sector of the economy. The second input is the proportion of such purchases that a central bank decides to reimburse or levy. Therefore, these novel monetary instruments can be seen as sector-specific rates on nominal consumption expenditure. Although these rates can be both positive and negative, in the remainder of the paper we would refer to them as cashback rates. Positive cashback rates correspond to a transfer from the central bank to the household and are financed by a monetary expansion (i.e. by expansion of the central bank's balance sheet). The opposite applies to the negative rates. From an economic perspective, the difference in design between this system of transfers and conventional interest rate policy would sum up to directly shifting demand for goods produced in a given sector (that is in need of an intervention) as opposed to influencing the aggregate demand for all goods. Therefore, the approach laid out in this paper is different in its mechanics from the conventional interest rate-based policy that operates through the wholesale money market aiming to shift the household's stochastic discount factor.

Although transmission of the policy intervention is different, both the cashback instrument and the interest rate operate by shifting demand: aggregate demand in case of the interest rate and relative demand in the case of the cashback instrument. Consider, for simplicity, a case in which firms produce goods using a production function that is linear in labour. If all firms operating in one sector of the economy are hit by a common positive productivity shock, the level of wages and, hence, marginal costs will fall on impact. As a consequence, price level will fall and relative demand for the goods produced in this sector will rise, leading to an increase in output. Since the cashback rate introduces a wedge between prices faced by consumers and those charged by producers, a central bank's commitment to a rule that increases the cashback rate when producer prices fall would produce a different equilibrium outcome. Higher cashback rate would further increase relative demand for goods produced in this sector as consumers

[6] A forthcoming paper by Aligishiev and Skovorodov (Forthcoming) evaluates the welfare costs of sticky information in economic surveillance of inflation targeting countries.

can now afford to buy greater quantities of goods at the expense of the monetary authority. Higher relative demand boosts production and increases labour demanded by firms. Resulting increase in wages, pushes marginal costs up and lifts prices set by producers. Therefore, if firms anticipate such an intervention and believe in central bank's commitment to such a policy they may reduce the magnitudes of price adjustments.

Carefully designed cashback policies can complement existing conventional monetary policy in two important ways. First, a crucial side effect of the above setup is that the cashback rate is positively related to the inflation rate; thus, there would be an effective lower bound on deflationary policies, not inflationary ones as is currently the case. A monetary authority can increase cashback rates to stimulate the economy without facing constraints when short-term interest rates are in close proximity to the ZLB. A crucial property keeping in mind the reduction in monetary policy space among advanced economies depicted in Figure 1.

Second, in a multi-sectoral setting under asymmetric shocks, cashback rates provide the monetary authority with additional instruments necessary to get closer to the efficient allocation of resources. As we will show in section IV, a combination of an optimal simple implementable interest and cashback rate rules systematically delivers a welfare gain compared to a case in which the interest rate rule is not supported. Sector-specific cashback rates are better suited to penalise deviations of average sectoral markups from their steady state levels as well as divergence relative to each other that opens the terms of trade gap. Hence, dispersion in markups is lower and households receive consumption basket that corresponds to a higher level of utility.

The next section describes the two-sector New Keynesian model that was used to achieve the aforementioned result.

IV Basic two-sector New Keynesian Model with cashback payments

The model is an extension of a single-sector New Keynesian model in Yun (2005) that is extended to accommodate two production sectors, labour mobility between sectors, and cashback payments. We preserve price-stickiness in both sectors to allow a non-trivial monetary policy problem in which a monetary authority cannot fully minimise distortions by means of the simple interest rate rule. Our approach follows more recent two-sector New Keynesian models in spirit while maintaining a comparatively simpler setup that would allow us to deliver a succinct investigation into the welfare properties of the proposed cashback instruments. As shown in Benigno (2004), keeping both sectors of the economy sticky often entails a trade-off in stabilising sectoral inflation and a term of trade gap. Papers in this vector of research often rely on picking optimal weights for sectoral price or wage inflation rates, that depend on assumptions imposed on the analysed economy. Such assumptions include varying degrees of price, presence of durable goods, labour mobility, etc.

In contrast to the above approach, we seek to utilise a basic model that is approximated around a non-distorted static equilibrium and disregards most of the complex modelling features (often implemented in the modern Dynamic Stochastic General Equilibrium (DSGE) models tailored for policy making) to show that introduction of cashback rules can achieve levels of welfare unattainable by means of the interest rate rule alone, however complicated.

We augment the New Keynesian model to allow for two types of prices: those faced by consumers when purchasing goods and those charged by firms when selling them. Keeping in mind the limited nature of the analysed DSGE model, we can comfortably denote these as consumer and producer prices. Cashback payments, initiated by a central bank, drive a wedge between these prices. In absence of cashback payments, these two types of prices converge to a singular price distribution customary used in the New Keynesian literature.

A. The representative household

A representative household receives utility from consumption $\{X_t\}$ and dis-utility from labour $\{N_t\}$. The period utility function $U(X_t, N_t)$ is assumed to be continuous and twice differentiable, with $U_{X,t} > 0$, $U_{XX,t} \leq 0$, $U_{N,t} \leq 0$, and $U_{NN,t} \leq 0$. The lifetime utility of an infinitely living household $\{\mathcal{W}_t\}$ is given by a discounted stream of period utilities:

$$\mathcal{W}_t = E_0 \sum_{t=0}^{\infty} \zeta_t \beta^t U\left(X_t, N_t\right)$$

where $\beta \in (0,1)$ is the subjective discount factor, E_0 is the expectation operator, ζ_t is a preference shock that follows an AR(1) process in logs, N_t the total hours worked by the household

$$N_t = \left[(1-\gamma)^{-\frac{1}{\rho}} \left(N^s_{A,t}\right)^{\frac{1+\rho}{\rho}} + \gamma^{-\frac{1}{\rho}} \left(N^s_{B,t}\right)^{\frac{1+\rho}{\rho}}\right]^{\frac{\rho}{1+\rho}}$$

and X_t the composite index of consumption

$$X_t = \frac{C_{A,t}^{1-\phi} C_{B,t}^{\phi}}{(1-\phi)^{1-\phi} \phi^{\phi}}$$

with $C_{A,t}$ and $N_{A,t}^s$ representing consumption of the composite sector A good and labour supplied to sector A, respectively. $C_{B,t}$ and $N_{A,t}^s$ have analogous definitions for sector B. $\phi \in [0,1]$ governs the relative weight of sector-specific consumption in the final consumption basket. $\gamma \in [0,1]$ represents the relative weight of sectoral labour supply in total labour supplied by the representative household. $\rho > 0$ represents the intra-temporal elasticity of substitution of labour across sectors. The composite index formulation assumes a unit elasticity of substitution between consuming goods produced in sectors A and B. $C_{A,t}$ and $C_{B,t}$ are given by Dixit-Stiglitz aggregates

$$C_{A,t} = \left(\int_0^1 C_{i,t}^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$$

and

$$C_{B,t} = \left(\int_0^1 C_{j,t}^{\frac{\eta'-1}{\eta'}} dj \right)^{\frac{\eta'}{\eta'-1}}$$

with $C_{i,t}$ and $C_{j,t}$ representing the quantities of goods $i \in [0,1]$ and $i \in [0,1]$, respectively, that are consumed by the household. Parameters η and η' govern the elasticity of substitution across different varieties of intermediate goods produced in sector A and sector B, respectively.

The monetary authority can directly affect prices, as faced by the representative household. It can either diminish the price of a given good by reimbursing household's nominal consumption expenditure on it or inflate its price via a levy on such expenditure.[7] The shift from subsidising to taxing consumption expenditure (and from deflating to inflating the price of a good) is captured by the change in the sign of the cashback rate $\{-1 < \Xi_{k,t} < 1, \text{ for } k = A, B\}$. A positive cashback rate would lead to partial reimbursement of consumption expenditure, similar in nature to the common practice of private sector financial institutions, while the negative cashback would artificially raise the price of a good above the level that would result from the market forces at play.

In this model, the monetary authority picks two cashback rates: one rate for goods produced in sector A and another one for those produced in sector B. In other words, all expenditure on goods produced in a given sector is subject to a common cashback rate. Therefore, individual

[7] Both scenarios imply a transfer of resources between the household and the monetary authority that, in reality, would be designed as a monetary transfer between a central bank and a household's CBDC wallet.

prices faced by the representative consumer can be represented as a wedge over the individual prices charged by producers operating in respective sectors:

$$P_{i,t}^{cons} = (1 - \Xi_{A,t})\, P_{i,t}^{prod} \tag{1}$$

and

$$P_{j,t}^{cons} = (1 - \Xi_{B,t})\, P_{j,t}^{prod} \tag{2}$$

where $P_{i,t}^{cons}$ is the end price of good i (Sector A) as faced by the consumer and $P_{i,t}^{prod}$ is the price of good i that was charged and received by the producer. Definitions of variables with subscript j (Sector B) follow suit.

Equations (1) and (2) rely on immediate clearance of cashback payments, not an unrealistic assumption keeping in mind the technological potential of a modern central bank to perform real-time operations with digital assets. By way of example, a $P_{i,t}^{prod}C_{i,t}$ amount spent on a good produced by firm i that operates in sector A would trigger a cashback payment of $\Xi_{A,t}P_{i,t}^{prod}C_{i,t}$ that supplements the household's income immediately. This setup allows us to represent the household budget constraint in terms of end prices faced by the representative consumer

$$\int_0^1 P_{i,t}^{cons}C_{i,t}di + \int_0^1 P_{j,t}^{cons}C_{j,t}dj + Q_{t,t+1}B_t \leq B_{t-1} + W_tN_t - T_t + \Omega_{A,t} + \Omega_{B,t} \quad \text{for } \ t = 0, 1, 2, \ldots$$

where B_t represents the quantity of one period nominal bonds purchased in period t at price $Q_{t,t+1}$ and maturing in period $t+1$, W_t denotes the nominal wage per unit of labour, T_t is a nominal lump-sum tax, and $\Omega_{A,t}$ and $\Omega_{B,t}$ are total dividends paid by firms operating in sector A and B, respectively. It is useful to keep in mind that the risk-free interest rate is implied by the bond market, such that:

$$R_t = \frac{1}{Q_{t,t+1}} \tag{3}$$

The optimal allocation of consumption among the infinite intermediary goods on both sectors is given by

$$\frac{C_{i,t}}{C_{A,t}} = \left[\frac{P_{i,t}^{cons}}{P_{A,t}^{CPI}}\right]^{-\eta} \qquad \text{for } \ i \in [0,1] \tag{4}$$

and

$$\frac{C_{j,t}}{C_{B,t}} = \left[\frac{P_{j,t}^{cons}}{P_{B,t}^{CPI}}\right]^{-\eta'} \qquad \text{for } \ j \in [0,1] \tag{5}$$

where the consumer price indexes for sectors A and B are given by

$$P_{A,t}^{CPI} = \left(\int_0^1 \left(P_{i,t}^{cons} \right)^{1-\eta} di \right)^{\frac{1}{1-\eta}} \tag{6}$$

and

$$P_{B,t}^{CPI} = \left(\int_0^1 \left(P_{j,t}^{cons} \right)^{1-\eta'} dj \right)^{\frac{1}{1-\eta'}} \tag{7}$$

Optimal intersectoral consumption choices are not affected by the cashback policy since all goods produced in a given sector are subject to the same cashback rate. On the other hand, optimal intersectoral allocation of consumption between sectors A and B is affected.

Optimal allocation of consumption between the two sectors is given by

$$\frac{C_{A,t}}{X_t} = (1-\phi) \left(\frac{P_{A,t}^{CPI}}{P_t^{HICP}} \right)^{-1} \tag{8}$$

and

$$\frac{C_{B,t}}{X_t} = \phi \left(\frac{P_{B,t}^{CPI}}{P_t^{HICP}} \right)^{-1} \tag{9}$$

where the HICP measure has the following representation

$$P_t^{HICP} = \left(P_{A,t}^{CPI} \right)^{1-\phi} \left(P_{B,t}^{CPI} \right)^{\phi} \tag{10}$$

It is important at this stage to acknowledge that the harmonised index of producers' prices (HIPP) follows the same definition as the HICP:

$$P_t^{HICP} = \left(P_{A,t}^{PPI} \right)^{1-\phi} \left(P_{B,t}^{PPI} \right)^{\phi} \tag{11}$$

Optimal allocation of labour between two sectors is obtained in a similar fashion. Under the assumption of a common labour market, optimal labour ratios are given by

$$\frac{N_{A,t}^s}{N_t} = (1-\gamma) \left(\frac{W_{A,t}}{W_t} \right)^{\rho} \tag{12}$$

and

$$\frac{N_{B,t}^s}{N_t} = \gamma \left(\frac{W_{B,t}}{W_t} \right)^{\rho} \tag{13}$$

The real wage index is then given by

$$\frac{W_t}{P_t^{HICP}} = \left[(1-\gamma)\left(\frac{W_{A,t}}{P_t^{HICP}}\right)^{1+\rho} + \gamma\left(\frac{W_{B,t}}{P_t^{HICP}}\right)^{1+\rho}\right]^{\frac{1}{1+\rho}} \tag{14}$$

Finally, under the optimal allocation of consumption, expressed above, the budget constraint can be represented as

$$P_t^{HICP}X_t + Q_tB_t \leq B_{t-1} + W_tN_t - T_t + \Omega_{A,t} + \Omega_{B,t} \quad \text{for } t = 0,1,2,\ldots \tag{15}$$

where the following optimal relationships were used:

1. $P_t^{HICP}X_t = P_{A,t}^{CPI}C_{A,t} + P_{B,t}^{CPI}C_{B,t}$;
2. $P_{A,t}^{CPI}C_{A,t} = \int_0^1 P_{i,t}^{cons}C_{i,t}di$;
3. $P_{B,t}^{CPI}C_{B,t} = \int_0^1 P_{j,t}^{cons}C_{j,t}dj$;
4. $W_tN_t = W_{A,t}N_{A,t}^s + W_{B,t}N_{B,t}^s$;

Maximising life-time utility subject to (15) and the transversality condition $\lim_{T\to\infty} E_t\{B_T\} \geq 0$ yields conventional static optimality condition and Euler consumption equation:

$$-\frac{U_{N,t}}{U_{X,t}} = \frac{W_t}{P_t^{HICP}} \tag{16}$$

$$Q_{t,t+1} = \beta\frac{\mathbb{E}_t\{\zeta_{t+1}U_{X,t+1}\}}{\zeta_tU_{X,t}}\frac{P_t^{HICP}}{\mathbb{E}_t\{P_{t+1}^{HICP}\}} \tag{17}$$

B. Firms

Each production sector is populated by a continuum of firms that operate under monopolistic competition. A typical firm hires labour from the representative household to produce output subject to sector-specific factor productivity shock. Therefore, firm-level output of sectors A and B is given by

$$Y_{i,t} = z_{A,t}N_{i,t}^d \tag{18}$$

and

$$Y_{j,t} = z_{B,t}N_{j,t}^d \tag{19}$$

where $Y_{i,t}$ represent the output of firm $i \in [0,1]$ operating in sector A and $Y_{j,t}$ that of firm $j \in [0,1]$ that operates in sector B, $N_{i,t}^d$ and $N_{j,t}^d$ are firm-level labour input, i.e. labour demanded by firms operating in these sectors. $z_{A,t}$ and $z_{B,t}$ represent sector-specific productivity shocks that follow AR(1) processes in logs.

A typical firm operating in sector A picks the labour demanded to minimise the cost of production subject to the production function (18). Assuming common labour market within each sector, the solution to such a problem is given by the following real marginal cost function

$$MC_{A,t} = \frac{(1-\tau_A)}{z_{A,t}} \frac{W_{A,t}}{P_{A,t}^{PPI}} \quad \text{for} \;\; t = 1, 2, \ldots \tag{20}$$

and its analogue in case of sector B

$$MC_{B,t} = \frac{(1-\tau_B)}{z_{B,t}} \frac{W_{B,t}}{P_{B,t}^{PPI}} \quad \text{for} \;\; t = 1, 2, \ldots \tag{21}$$

where τ_A and τ_B are production subsidies financed by lump-sum taxes. Using (1), (2), (6), and (7) we can represent sectoral producer price indexes $P_{A,t}^{PPI}$ and $P_{B,t}^{PPI}$ as

$$P_{A,t}^{PPI} = \frac{P_{A,t}^{CPI}}{(1-\Xi_{A,t})} \tag{22}$$

and

$$P_{B,t}^{PPI} = \frac{P_{B,t}^{CPI}}{(1-\Xi_{B,t})} \tag{23}$$

Although inside a given sector, all firms face the same marginal costs, firms in different sectors do not, due to potentially different realisations of sector-specific technology shocks.

Staggered prices are modelled following Calvo (1983) with both sectors assumed to be sticky. Each period a constant fraction $1-\theta$ of firms operating in sector A gets to pick a new price $\{P_t^*\}$ (where the subscript i can be omitted since all re-optimising firms will choose the same new price) for the goods they produce. Analogously, θ' and $\tilde{P}_t$ are the stickiness parameter and the new optimal price chosen in case of sector B. Firms that get a chance to set a new price in period t seek to maximise current market value of future profits discounted by θ, subject to the sequence of demand schedules $Y_{i,t+k|t} = (1-\phi)\left(\frac{P_{i,t}^{cons}}{P_{A,t+k}^{CPI}}\right)^{-\eta}\left(\frac{P_{A,t+k}^{CPI}}{P_{t+k}^{HICP}}\right)^{-1} X_{t+k}$ for $k = 0, 1, 2, \ldots$ (where the subscript i is replaced with a superscript a below, since all the firms in sector A that reset their price in period t will face the same sequence of demand schedules) and the prevailing cashback rate $P_{i,t}^{cons} = (1-\Xi_{A,t})P_t^*$. Using (22), the solution to the price-setting problem has a conventional for New Keynesian literature form:

$$\frac{P_t^*}{P_{A,t}^{PPI}} = \frac{\sum_{k=0}^{\infty} (\theta\beta)^k \, \mathbb{E}_t \Big\{ \mathcal{M}_A ms_{A,t+k} (1-\phi) X_{t+k} U_{X,t+k} MC_{A,t+k} \left(\Pi_{A,t,t+k}^{PPI}\right)^{\eta+1} \left(\Pi_{A,t,t+k}^{CPI}\right)^{-1} \Big\}}{\sum_{k=0}^{\infty} (\theta\beta)^k \, \mathbb{E}_t \Big\{ (1-\phi) X_{t+k} U_{X,t+k} \left(\Pi_{A,t,t+k}^{PPI}\right)^{\eta} \left(\Pi_{A,t,t+k}^{CPI}\right)^{-1} \Big\}} \tag{24}$$

where $\mathcal{M}_A \equiv \frac{\eta}{\eta-1}$ is a constant flexible-price markup desired by firms operating in sector A, $\Pi_{A,t,t+k}^{PPI} = \frac{P_{A,t+k}^{PPI}}{P_{A,t}^{PPI}}$ is the sector-specific PPI inflation rate between $t+k$ and $t-1$. $ms_{A,t}$ represents a markup shock that hits sector A exclusively, defined as an ARMA(1,1) process in logs. By analogy, a solution to the price-setting problem in sector B is given by

$$\frac{\tilde{P}_t}{P_{B,t}^{PPI}} = \frac{\sum_{k=0}^{\infty} \left(\theta'\beta'\right)^k \, \mathbb{E}_t \Big\{ \mathcal{M}_B ms_{B,t+k} \phi X_{t+k} U_{X,t+k} MC_{B,t+k} \left(\Pi_{B,t,t+k}^{PPI}\right)^{\eta'+1} \left(\Pi_{B,t,t+k}^{CPI}\right)^{-1} \Big\}}{\sum_{k=0}^{\infty} \left(\theta'\beta'\right)^k \, \mathbb{E}_t \Big\{ \phi X_{t+k} U_{X,t+k} \left(\Pi_{B,t,t+k}^{PPI}\right)^{\eta'} \left(\Pi_{B,t,t+k}^{CPI}\right)^{-1} \Big\}} \tag{25}$$

C. Market clearing

Market clearing in goods markets implies that all goods produced by firm i are consumed by the representative household:

$$Y_{i,t} = C_{i,t}$$
$$Y_{j,t} = C_{j,t}$$

and, therefore, the firm-specific production is aggregated to the sector level using the Dixit-Stiglitz function described above:

$$Y_{A,t} = \left(\int_0^1 Y_{i,t}^{1-\frac{1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} = C_{A,t} \tag{26}$$

$$Y_{B,t} = \left(\int_0^1 Y_{j,t}^{1-\frac{1}{\eta'}} dj \right)^{\frac{\eta'}{\eta'-1}} = C_{B,t} \tag{27}$$

Finally, real aggregate output of the economy equals to the sum of sectoral outputs re-scaled using respective relative prices.

$$Y_t = \frac{P_{A,t}^{CPI}}{P_t^{HICP}} Y_{A,t} + \frac{P_{B,t}^{CPI}}{P_t^{HICP}} Y_{B,t} \tag{28}$$

Market clearing on the labour market implies that the sum of firm-specific labour demands equals to the aggregate sector-specific labour demand:

$$N_{A,t}^d = \int_0^1 N_{i,t}^d di$$
$$N_{B,t}^d = \int_0^1 N_{j,t}^d dj$$

Furthermore, it is assumed that the representative household always supplies enough labour to cover sector-specific demands, i.e. the labour market clears:

$$N_{A,t}^d = N_{A,t}^s \tag{29}$$

$$N_{B,t}^d = N_{B,t}^s \tag{30}$$

D. Equilibrium

HIPP inflation is given by the HIPP index (11):

$$\Pi_t^{HIPP} = \left(\Pi_{A,t}^{PPI}\right)^{(1-\phi)} \left(\Pi_{B,t}^{PPI}\right)^{\phi} \tag{31}$$

while the sectoral PPI inflation rates can be obtained using (1),(2),(4),(5),(6), and (7):

$$1 = \theta \left(\Pi_{A,t}^{PPI}\right)^{\eta-1} + (1-\theta)\, p_t^{*1-\eta} \tag{32}$$

$$1 = \theta' \left(\Pi_{B,t}^{PPI}\right)^{\eta'-1} + \left(1-\theta'\right) \tilde{p}_t^{1-\eta'} \tag{33}$$

where $p_t^* = \frac{P_t^*}{P_{A,t}^{PPI}}$ and $\tilde{p}_t = \frac{\tilde{P}_t}{P_{B,t}^{PPI}}$.

By analogy, we derive the HICP inflation rate

$$\Pi_t^{HICP} = \left(\Pi_{A,t}^{CPI}\right)^{(1-\phi)} \left(\Pi_{B,t}^{CPI}\right)^{\phi} \tag{34}$$

where using (22) and (23) we obtain sectoral CPI inflation rates

$$\Pi_{A,t}^{CPI} = \frac{1-\Xi_{A,t}}{1-\Xi_{A,t-1}} \Pi_{A,t}^{PPI} \tag{35}$$

and

$$\Pi_{B,t}^{CPI} = \frac{1-\Xi_{B,t}}{1-\Xi_{B,t-1}} \Pi_{B,t}^{PPI} \tag{36}$$

Using (4), (5), and the goods market clearing conditions we obtain sectoral outputs:

$$\Delta_{A,t} Y_{A,t} = z_{A,t} N_{A,t}^{d} \tag{37}$$

$$\Delta_{B,t} Y_{B,t} = z_{B,t} N_{B,t}^{d} \tag{38}$$

where $\Delta_{A,t} = \int_0^1 \left(\frac{P_{i,t}^{cons}}{P_{A,t}^{CPI}} \right)^{-\eta} di$ and $\Delta_{B,t} = \int_0^1 \left(\frac{P_{j,t}^{cons}}{P_{B,t}^{CPI}} \right)^{-\eta'} dj$ represent relative price dispersion measures within each sector. Under the Calvo setup, these dispersion terms can be shown to evolve according to

$$\Delta_{A,t} = (1-\theta) \left(p_t^*\right)^{-\eta} + \theta \Delta_{A,t-1} \left(\Pi_{A,t}^{CPI}\right)^{\eta} \tag{39}$$

and

$$\Delta_{B,t} = (1-\theta') \left(\tilde{p}_t\right)^{-\eta'} + \theta' \Delta_{B,t-1} \left(\Pi_{B,t}^{CPI}\right)^{\eta'} \tag{40}$$

By defining $J_{A,t}$ and $JJ_{A,t}$ as the numerator and denominator of the right-hand side in (24), it can be represented recursively as:

$$p_t^* = \frac{J_{A,t}}{JJ_{A,t}} \tag{41}$$

$$J_{A,t} = \mathcal{M}_A ms_{A,t} (1-\phi) X_t U_{X,t} MC_{A,t} + \theta \beta \mathbb{E}_t \left\{ \frac{\left(\Pi_{A,t+1}^{PPI}\right)^{\eta+1}}{\Pi_{A,t+1}^{CPI}} J_{A,t+1} \right\} \tag{42}$$

$$JJ_{A,t} = (1-\phi) X_t U_{X,t} + \theta \beta \mathbb{E}_t \left\{ \frac{\left(\Pi_{A,t+1}^{PPI}\right)^{\eta}}{\Pi_{A,t+1}^{CPI}} JJ_{A,t+1} \right\} \tag{43}$$

where (3) was used. By analogy, solution to a price-setting problem for sector B (25) can be represented as:

$$\tilde{p}_t = \frac{J_{B,t}}{JJ_{B,t}} \tag{44}$$

$$J_{B,t} = \mathcal{M}_B ms_{B,t} \phi X_t U_{X,t} MC_{B,t} + \theta' \beta' \mathbb{E}_t \left\{ \frac{\left(\Pi^{PPI}_{B,t+1}\right)^{\eta'+1}}{\Pi^{CPI}_{B,t+1}} J_{B,t+1} \right\} \tag{45}$$

$$JJ_{B,t} = \phi X_t U_{X,t} + \theta' \beta' \mathbb{E}_t \left\{ \frac{\left(\Pi^{PPI}_{B,t+1}\right)^{\eta'}}{\Pi^{CPI}_{B,t+1}} JJ_{B,t+1} \right\} \tag{46}$$

Using the CPI (10) and defining the relative price as $q_t \equiv \frac{P^{PPI}_{A,t}}{P^{PPI}_{B,t}}$, we can represent (8), (9), (20), (21), and (28) as:

$$\frac{C_{A,t}}{X_t} = (1-\phi) \left[\frac{1-\Xi_{A,t}}{1-\Xi_{B,t}} q_t \right]^{-\phi} \tag{47}$$

$$\frac{C_{B,t}}{X_t} = \phi \left[\frac{1-\Xi_{A,t}}{1-\Xi_{B,t}} q_t \right]^{1-\phi} \tag{48}$$

$$MC_{A,t} = \frac{(1-\tau_A)}{z_{A,t}} \frac{W_{A,t}}{P^{HICP}_t} (1-\Xi_{A,t})^{1-\phi} (1-\Xi_{B,t})^{\phi} q_t^{-\phi} \tag{49}$$

$$MC_{B,t} = \frac{(1-\tau_B)}{z_{B,t}} \frac{W_{B,t}}{P^{HICP}_t} (1-\Xi_{A,t})^{1-\phi} (1-\Xi_{B,t})^{\phi} q_t^{1-\phi} \tag{50}$$

$$Y_t = \left[\frac{1-\Xi_{A,t}}{1-\Xi_{B,t}} q_t \right]^{\phi} Y_{A,t} + \left[\frac{1-\Xi_{A,t}}{1-\Xi_{B,t}} q_t \right]^{\phi-1} Y_{B,t} \tag{51}$$

where q_t is given from $\frac{P^{PPI}_{A,t}}{P^{PPI}_{B,t}} \equiv \frac{P^{PPI}_{A,t}}{P^{PPI}_{A,t-1}} \frac{P^{PPI}_{A,t-1}}{P^{PPI}_{B,t-1}} \frac{P^{PPI}_{B,t-1}}{P^{PPI}_{B,t}}$, or alternatively:

$$q_t = q_{t-1} \frac{\Pi^{PPI}_{A,t}}{\Pi^{PPI}_{B,t}} \tag{52}$$

Finally, real value of retail CBDC in circulation $\{M^{cbdc}_t\}$ is given by:

$$M^{cbdc}_t = M^{cbdc}_{t-1} + \Xi_{A,t} C_{A,t} + \Xi_{B,t} C_{B,t} \tag{53}$$

A competitive equilibrium of the sticky-price economy is given by the set of processes Y_t, $Y_{A,t}$, $Y_{B,t}$, X_t, $C_{A,t}$, $C_{B,t}$, N_t, $N^s_{A,t}$, $N^s_{B,t}$, $N^d_{A,t}$, $N^d_{B,t}$, $\frac{W_t}{P^{HICP}_t}$, $\frac{W_{A,t}}{P^{HICP}_t}$, $\frac{W_{B,t}}{P^{HICP}_t}$, q_t, R_t, Π^{HICP}_t, $\Pi^{CPI}_{A,t}$, $\Pi^{CPI}_{B,t}$, Π^{HIPP}_t, $\Pi^{PPI}_{A,t}$, $\Pi^{PPI}_{B,t}$, $MC_{A,t}$, $MC_{B,t}$, p^*_t, $\tilde{p}_t$, $J_{A,t}$, $JJ_{A,t}$, $J_{B,t}$, $JJ_{B,t}$, $\Delta_{A,t}$,$\Delta_{B,t}$, $\Xi_{A,t}$, $\Xi_{B,t}$, M^{cbdc}_t for $t = 0, 1, ...$ that remain bounded in some neighbourhood of a deterministic steady-state and satisfy equations: (12)-(14), (16)-(17), (26)-(27), (29)-(53), an interest rate rule, two cashback rate rules, and given initial values of q_{-1}, B_{-1}, M^{cbdc}_{-1} exogenous stochastic and functional forms. Further details can be found in Appendix B.

V Zero lower bound

One of the most obvious applications of the cashback instruments is to supplement conventional monetary policy when the policy rate hits the ZLB. Due to the nature of cashbacks payments, such instruments have a lower bound on deflationary policies, not inflationary ones.[8] Thus, it is possible to stimulate economic activity by introducing cashback payments even when interest rates cannot be lowered further. Moreover, a prompt use of cashback stimulus, as will be shown below, may allow a monetary authority to avoid the ZLB altogether under certain assumptions regarding policy targets.

In order to demonstrate the usefulness of cashbacks in such instances, we introduce an endogenous ZLB duration that is triggered by a negative preference shock. Following Christiano et al. (2011), the nominal interest rate is then determined by:

$$R_t = max\left(R_t^{policy}, \epsilon\right) \tag{54}$$

where the policy rate is given by a simple and implementable interest rate rule:

$$log\left(\frac{R_t^{policy}}{\bar{R}^{policy}}\right) = \rho_r log\left(\frac{R_{t-1}^{policy}}{\bar{R}^{policy}}\right) + \rho_\pi log\left(\frac{\Pi_t^{target}}{\Pi^{t\bar{a}rget}}\right) + \rho_y log\left(\frac{y_t}{\bar{y}}\right) \tag{55}$$

and ϵ is a parameter calibrated to represent the gross interest rate level at which the lower bound becomes binding. Importantly, following Cantelmo and Melina (2017), the interest rate rule reacts to a harmonised inflation measure that combines inflation rates observed in the two sectors using some weight τ:[9]

$$\Pi_t^{target} = \left(\Pi_{A,t}^{PPI}\right)^{(1-\tau)} \left(\Pi_{B,t}^{PPI}\right)^{\tau}$$

Since in this section we are interested in the effects of unanticipated policy shocks as opposed to the consequences of introducing systematic policy rules, cashback rates follow AR(1)

[8] Higher cashback rates incentivise a higher relative use of CBDC for transaction purposes as compared to cash.

[9] The analysis in this section assumes $\epsilon = 1.0012$, $\theta = \theta' = 0.75$ $\rho_R = 0.5$, $\rho_\pi = 23.55$, $\rho_y = 0$, and $\tau = 0.5$. The high value of the inflation reaction parameter is taken from the welfare-maximising rule in the next section, corresponding to the chosen level of price stickiness.

processes:

$$\Xi_{n,t} = \rho_n^{\xi} \Xi_{n,t-1} + \varepsilon_t^{\xi,n} \quad \text{for} \quad n \in \{A, B\}$$

A surprise increase in cashback rates in both sectors increases the purchasing power of the representative household's income as real wages rise in both sectors. A resulting shift in the aggregate demand for goods and services leads to a higher level of consumption and, therefore, higher output. In turn, facing higher demand, producers do not need to decrease their prices as much as they would have in absence of the cashback stimulus. As a result, the possibility of a deflationary spiral is minimised, if consumers expect higher real wages, induced by the government stimulus, to last until the shock causing a recession is dissipated.

Let us now consider two examples of how cashback stimulus can respond to a negative preference shock, calibrated to deliver seven quarters of ZLB. First, an unanticipated one-off increase in the cashback rates that can soften the initial impact of a recession. Secondly, a similar surprise increase in cashback rates that is expected to gradually dissipate over time.

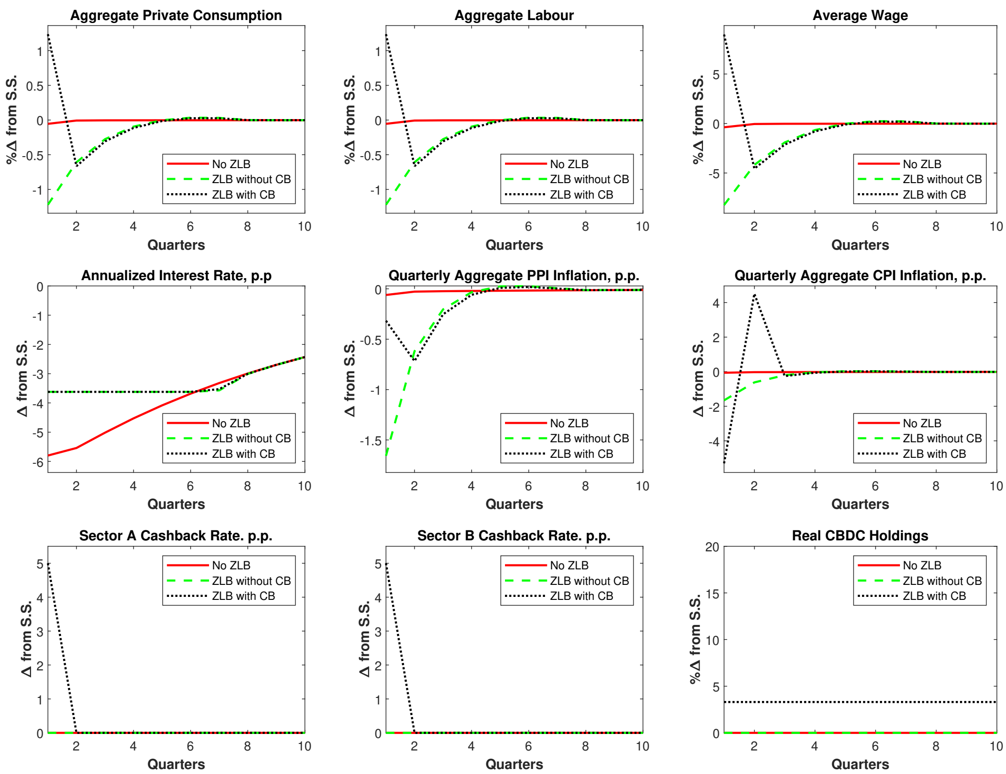

Figure 5: Impulse-response functions to a surprise temporary 5 percentage point shock to $\Xi_{a,t}$ and $\Xi_{B,t}$.

Figure 5 presents the first case. If the ZLB is binding (green dashed line) the economy experiences a larger fall in aggregate consumption and employment, a more pronounced fall in the average wage rate, and a stronger deflationary pressure than in the case when the interest rate is allowed to adjust freely (red solid line). A surprise one-off five percentage point increase in cashback rates (black dotted line) can successfully stimulate private consumption expenditure and employment on impact. The resulting one-period rise in aggregate demand reduces the deflationary pressure on producer prices. This effect is transitory and has a minimal effect on intertemporal allocation of consumption, since in its source lies a monetary-financed single-period boost to household income that cannot be carried over.[10] This can be further observed in Figure 7.a which plots price indices prevailing under the said stimulus over a single price index prevailing if cashback rates are set to zero. A one-off shock affects producer prices only instantaneously and has no impact on the consequent relative change in future prices.

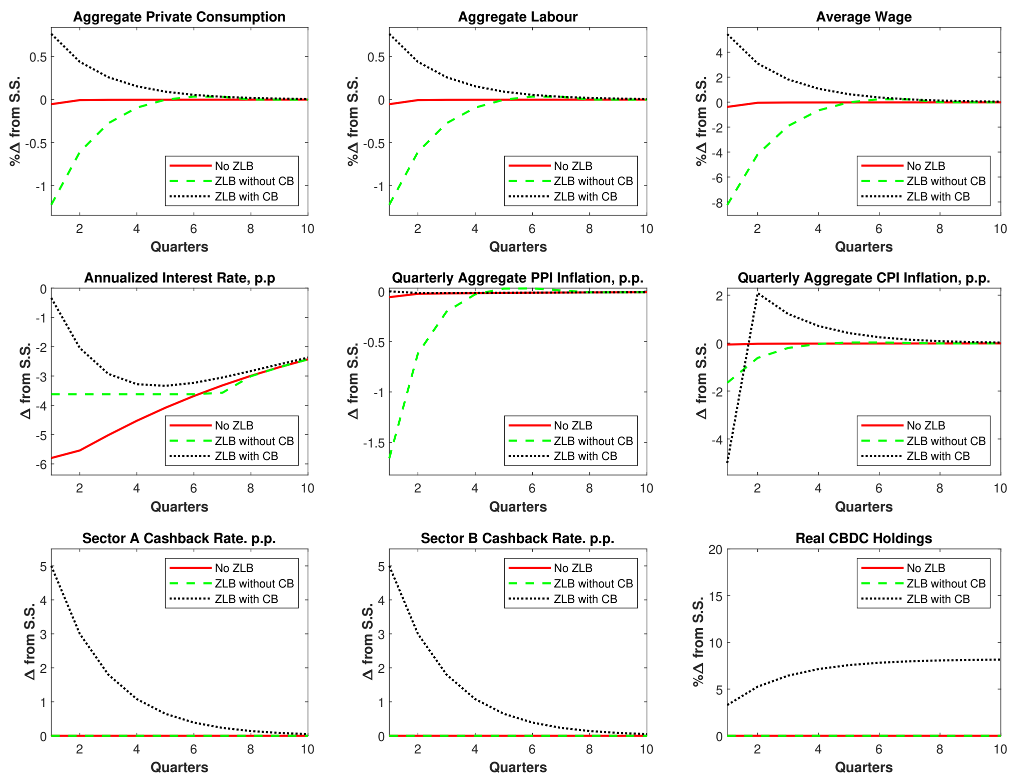

Figure 6: Impulse-response functions to a surprise temporary 5 percentage point shock to $\Xi_{a,t}$ and $\Xi_{B,t}$. Shock modelled as an AR(1) process with an autoregressive coefficient equal to 0.65

A more pronounced effect on the economic activity is delivered if the increase in the cashback rates is more persistent. A persistent increase in the cashback rate is calibrated to last

[10]Similar in nature to the U.S. stimulus checks but one that is instead linked to household consumption expenditure, i.e. a monetary financed stimulus that cannot be saved.

approximately ten quarters, gradually decreasing to the initial zero-cashback level.[11] Figure 6 compares such a case with the two benchmarks discussed above. In this case, the cashback rate stimulus delivers a greater impact on the aggregate demand in the medium term. Since households anticipate that the positive cashback rates will prevail for quite some time, aggregate consumption and employment never fall below the long-run level. As a result, the decrease in the producer prices is mild. Moreover, it is not caused by a decrease in demand, rather a decrease in the costs of production.

To elaborate on the last idea, a firm operating under monopolistic competition seeks to set its price as a fixed markup over nominal marginal costs. A positive economy-wide cashback leads to an increase in real level of wages *ceteris paribus*, which, in turn, implies that in a perfectly competitive labour market nominal wage would decrease. Lower nominal wages reduce the optimal price charged by the producer and, hence, lead to a deflation of producer prices that induces higher output.

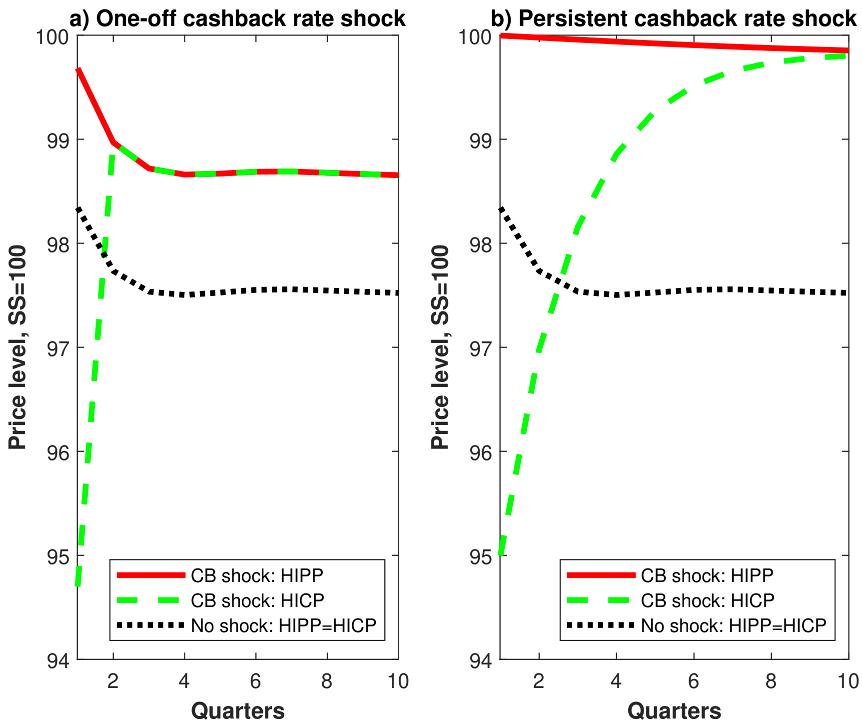

Figure 7: Response of price level measures to a preference shock. Black line demonstrates the baseline case of absent cashback policy.

As can be seen in Figure 6, a persistent cashback stimulus combined with a HIPP inflation target for the interest rate rule can help the monetary authority avoid the ZLB. Targeting the HIPP inflation is preferred since cashback policy operates through consumer prices, and, in this

[11] This is achieved by assuming $\rho_A^\xi = \rho_B^\xi = 0.6$.

formulation, will not produce any automatic offsetting policy interventions.

Regarding the size of the monetary expansion, our calibration provides modest estimates of the necessary expansion in the central bank's CBDC liabilities. A surprise one-off 5 percentage point increase in the cashback rate would expand the central bank's balance sheet by an additional 3.3 percent, while the persistent stimulus by a cumulative 8.2 percent by the tenth quarter.[12]

VI Normative analysis

This section seeks to determine if it is possible to achieve a welfare gain by complementing conventional monetary policy arsenal with cashback payments. In contrast to the policy rate intervention that, due to its design, delivers an economy-wide impact, cashback rates can be sector-specific and, thus, produce a welfare gain in a multi-sectoral setup that is subject to asymmetric shocks and varying degrees of nominal rigidities. In other words, cashback payments can be useful in countries or currency unions that are characterised by sufficiently heterogeneous regions, sectors, or member countries. By way of example, members of the Euro area experienced sufficiently heterogeneous developments in consumer price indices between 2019 and 2020 due to COVID-19 pandemic (Figures 3 and 4). Such heterogeneity could not be addressed by the policy rate alone since its ability to reduce dispersion in inflation rates is limited.

We construct the analysis around simple and implementable rules, with such rules defined broadly following Schmitt-Grohé and Uribe (2007), to produce results that can be used by a policymaker in a realistic setup, a setup in which natural rate of interest, output gap and flexible economy terms of trade are not observable. The aim of this section is to determine if the introduction of simple cashback rate rules can be welfare-inducing. In doing so we utilise the following two-step approach. First, we leverage the model to perform a parameter grid search on the simple implementable interest rate rule, aiming to maximise the representative household's lifetime welfare. Once we converge on the optimised interest rate rule, we perform a second joint grid search on the interest rate and two cashback rate rules to see if there exists a combination of the three that improves welfare relative to the results obtained in the first step.[13]

[12] For comparison, Federal Reserve's liabilities in the form of currency in circulation and deposits of depository institutions increased by almost 60 percent as a reaction to the first year of the COVID-19 pandemic (FED, 2021).

[13] The parameter grid search approach follows Schmitt-Grohé and Uribe (2007) and Cantelmo and Melina

The interest rate rule is given by:

$$log\left(\frac{R_t^{policy}}{\bar{R}^{policy}}\right) = \rho_r log\left(\frac{R_{t-1}^{policy}}{\bar{R}^{policy}}\right) + \rho_\pi log\left(\frac{\Pi_t^{target}}{\Pi^{\bar{target}}}\right) + \rho_y log\left(\frac{y_t}{y_{t-1}}\right)$$

where the policy rate reaction to the harmonised inflation rate measure is governed by ρ_π:

$$\Pi_t^{target} = \left(\Pi_{A,t}^{CPI}\right)^{(1-\tau)} \left(\Pi_{B,t}^{CPI}\right)^{\tau}$$

and reaction to the output growth of the entire economy is governed by ρ_y, subject to some degree of persistence in the policy rate given by ρ_r. The cashback rules are only allowed to react to the PPI inflation rate in the associated sector $\{\gamma_A^\pi, \gamma_B^\pi\}$:

$$\Xi_{n,t} = \gamma_n^\pi log\left(\frac{\Pi_{n,t}^{PPI}}{\Pi_n^{\bar{PPI}}}\right) \quad \text{for} \quad n \in \{A, B\}$$

In order to quantify the welfare gain of introducing cashback instruments we approximate household welfare and the entire set of equilibrium conditions of the previously described DSGE model to a second order. As customary in the literature, we present the compensating variation welfare metric in terms of a permanent change in consumption necessary to equate welfare levels between different policy formulations, given by

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[U\left(\left(1+\lambda^c\right)X_t, N_t\right)\right]\right\} = E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[U\left(\tilde{X}_t, \tilde{N}_t\right)\right]\right\} \tag{56}$$

where $\lambda^c \times 100$ represents the percent permanent increase in private consumption needed to make the household indifferent to the prospect of being subject to a more narrow monetary policy toolkit, $\{\tilde{X}_t, \tilde{N}_t\}$ for $t = 0, 1, ..., \infty$ are sequences of optimal consumption and labour choices in an economy with operational optimised cashback rate rules, and $\{X_t, N_t\}$ for $t = 0, 1, ..., \infty$ are such choices under the assumption that cashback rates were not implemented as policy instruments. As a shortcut, $\lambda^c > 0$ would imply that welfare level under the combination of interest and cashback rate rules is higher than that under the interest rate rule alone, and vice versa.

Table 1 presents results of the exercise described above, highlighting the parameter values corresponding to the implementable rules that maximise average welfare of the population. We

(2017). We define the support of parameters ρ^π and ρ^y to be $[0, 25]$, support of parameters ρ^R and τ to be $[0, 1]$, and support of parameters γ_A^π and γ_B^π to be $[-25, 0]$.

present welfare loss relative to the efficient allocation of resources as well as the compensation variation welfare metric between the two policies.[14] First, we consider a special case when both sectors are characterised by the same degree of price stickiness, be it a mild ($\theta = \theta' = 0.25$), moderate ($\theta = \theta' = 0.50$), or severe ($\theta = \theta' = 0.75$). Then, we look at a more general case when price stickiness in the two sectors is allowed to be different. Finally, we briefly demonstrate how the results change if one of the sectors is allowed to be flexible.[15]

Panel a:			$\theta = 0.75$	and	$\theta' = 0.75$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0	0	23.552	0.500	-	-	10.5581	-
IR+CBR	0.968	0	2.228	0.500	-3.485	-3.485	0.4525	6.327%
Panel b:			$\theta = 0.5$	and	$\theta' = 0.5$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0.069	0	8.382	0.500	-	-	2.3234	-
IR+CBR	0.936	0.048	1.636	0.500	-1.072	-1.072	0.0663	1.472%
Panel c:			$\theta = 0.25$	and	$\theta' = 0.25$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0	0	2.014	0.500	-	-	0.3495	-
IR+CBR	0.850	0.191	1.588	0.500	-0.347	-0.347	0.0081	0.228%
Panel d:			$\theta = 0.25$	and	$\theta' = 0.5$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0	0	2.208	0	-	-	0.4451	-
IR+CBR	0.823	0	4.489	0.001	-0.345	-2.008	0.0100	0.285%
Panel e:			$\theta = 0.25$	and	$\theta' = 0.75$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0	0	4.708	0	-	-	0.4320	-
IR+CBR	0.491	0.943	3.93	0.011	-0.343	-3.029	0.0083	0.278%
Panel f:			$\theta = 0.25$	and	$\theta' = 0$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0	0.006	2.127	1	-	-	0	-
IR+CBR	0	0.006	2.127	1	0	0	0	0%

Table 1: Optimised implementable rules and associated welfare gain. A case of asymmetric technology shocks.

Table 1 presents several notable observations. First, there is only room for a cashback policy

[14]The efficient allocation of resources, i.e. the flexible economy, assumes that all firms are free to adjust prices every period ($\theta = \theta' = 0$) and take prices as given. In other words, prices are flexible and firms cannot charge markups over marginal costs.

[15]All results in Table 1 assume two sectors of equal size and a unit labour mobility. To make results comparable with earlier literature, particularly with papers that leverage the linear-quadratic approach to welfare loss measurement (Benigno and Woodford, 2003) and those relying on Rotemberg (1982) menu costs to model nominal rigidities, we assume that price dispersion is not distortionary, i.e. $\Delta_{A,t} = \Delta_{B,t} = 1$.Furthermore, we focus on the case with asymmetric technology shocks only. Appendix D presents a robustness check of this section to a case in whch the analysed economy is hit by both asymmetric technology and markup shocks.

to be welfare-inducing if both sectors have nominal rigidities. A conventional implementable interest rate rule alone can successfully close the gap in household welfare between the sticky price economy and its efficient counterpart, if subjected to asymmetric technology shocks only, as can be seen in Panel f of Table 1. Alternatively, if both sectors are sticky, complementing a conventional interest rate rule with two sector-specific cashback rate rules systematically produces a welfare gain.

Secondly, if prices are sticky in both sectors, welfare gain from introducing cashback policy rules increases with the severeness of nominal rigidities. According to Panel a of Table 1, welfare gain accomplished by the introduction of cashback rules reaches as high as a permanent 6.3% increase in consumption expenditure when 75% of firms are stuck with their prices each period in both sectors.[16]

Thirdly, optimised implementable cashback rate rules imply a negative reaction of the cashback rates to sectoral inflation. A monetary authority should decrease cashback rates paid on expenditure in the sector where inflation is on the rise, thus, reducing the relative demand for goods produced in such sector.

Fourthly, if both sectors are characterised by the same degree of stickiness, reaction to sectoral inflation rates should be equalised across sectors. Moreover, the higher the degree of price stickiness, the larger should be the absolute value of the reaction parameter. By way of example, if the share of firms stuck with their old prices increases from 25% to 75%, the magnitude of the optimised reaction parameter rises from 0.3 to 3.5.

Finally, similar to the Benigno (2004) prescription for conventional monetary policy, if the two sectors have different degrees of price stickiness, the response to inflation in a more sticky sector should be prioritised (i.e. assigned a reaction coefficient with a higher magnitude) in formulating the cashback policy.

Keeping in mind these observations, we also seek to identify cases in which the welfare benefits of cashback rate adoption could be further maximised. In order to do so, we calculate how the compensating variation welfare metric changes with varying degree of labour mobility and relative sector sizes. We do so for the sector price stickiness combinations considered above. According to the Figure 8, we can further emphasise two observations.

[16]The reader should interpret results related to the compensating welfare differential with care. Our analysis aims to showcase that introduction of cashback rate rules can be welfare-inducing. To obtain somewhat realistic measure of the welfare gain, our analysis should be extended to incorporate all the "bells and whistles" of modern New Keynesian literature and accommodate Bayesian estimation of such a model on a credible set of data.

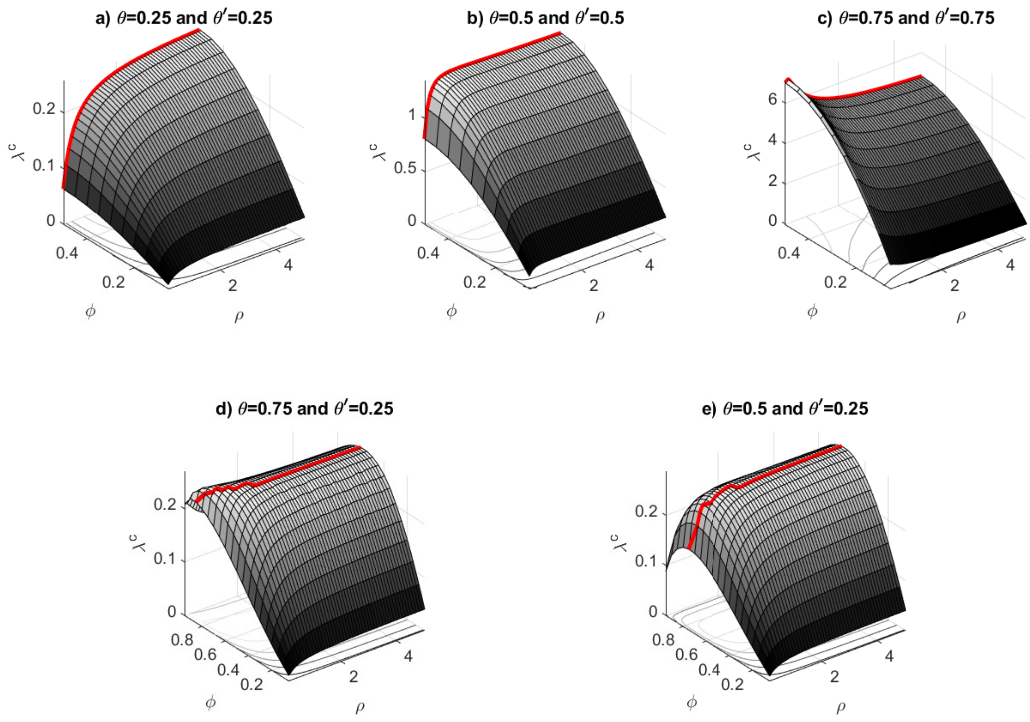

Figure 8: Welfare gain from adding cashback instruments to the monetary policy toolkit as a function of the sector size (ϕ) and labour mobility (ρ). Red line depicts parameter combinations that maximise the welfare gain in each case.

First, while the severeness of nominal rigidities is at most modest, higher mobility of labour implies higher welfare gain from introducing cashback instruments. When stickiness is higher, on the other hand, adoption of cashback instruments produces a higher welfare gain if labour mobility is low. This observation is not driven by the change in the welfare properties of cashback rate instruments, rather by the fact the implementable interest rate rule performs worse under the outlined conditions. Intuitively, such cases should be a natural choice for introduction of cashback rate rules.

Secondly, these symmetric cases imply that the welfare gain is maximised when sectors are of equal size. If price stickiness is not the same across sectors, welfare gain from the adoption of cashback rates is maximised when the more sticky sector constitutes a smaller proportion of the analysed economy. As before, this is driven by the worse performance of conventional monetary policy in certain cases.

VII Conclusion

This paper introduces cashback rates on private consumption expenditure to the monetary policy debate. We argue that cashback rates represent a valid complement to the conventional monetary policy toolkit in four important instances.

First, since cashback rates have a lower bound on deflationary policies, they are well suited to provide additional stimulus to the economy at times when conventional monetary policy is at the zero lower bound. In other words, a central bank can still stimulate aggregate demand and economic growth by increasing cashback rates, even when decreasing interest rates is no longer feasible. We show that if the interest rate rule targets an inflation rate derived using the harmonised index of producer prices, a persistent cashback rate stimulus provides the means for the monetary policy to avoid the zero lower bound and an associated fall in employment and output.

Secondly, unlike most of the tools in the conventional and unconventional monetary policy toolkits, cashback rates do not depend on the financial or corporate sectors to propagate the liquidity injections. These novel instruments can be seen as direct monetary-financed transfers to (or levies on) the household budgets. Importantly, these instruments are linked to consumption expenditure, and, thus, such stimulus cannot be set aside or saved. By way of example, our simulations show that a surprise one-off increase in the cashback rates will mostly stimulate aggregate demand on impact with only minor consequences for future consumption-savings choices of households.

Thirdly, sector-specific cashback rates can have a heterogeneous effect on sectoral or regional inflation rates, providing additional degrees of freedom that the current status quo of the monetary policy lacks. Since a central bank can come up with a cashback instrument for each sector or region of interest, these rates are better suited to counter consequences of asymmetric shocks, such as diverging price adjustments in demand-constrained and supply-constrained sectors observed during the COVID-19 pandemic.

Finally, the adoption of implementable cashback rate rules to support the conventional interest rate rule can bring a multi-sector sticky-price economy closer to the efficient allocation of resources. Most importantly, the welfare gain from introducing these rules is increasing in the overall stickiness of the economy. Additional setups in which the conventional interest rate rule underperforms are natural candidates for cashback rate adoption. By way of example, if the more sticky sector constitutes a smaller share of the economy or when labour is substantially mobile but prices are substantially sticky.

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Appendices

A Calibration

Parameter		Value
Discount factor	β	0.99
Labour preference parameter	ν	5
Risk aversion	σ	2
Labour mobility parameter	ρ	1
Share of sector B goods in consumption basket	ϕ	0.5
Share of labour supplied to sector B	γ	0.5
Elasticity of substitution in Sector A of the goods market	η	9
Elasticity of substitution in Sector B of the goods market	η'	9
Calvo price stickiness, Sector A	θ	0.25
Calvo price stickiness, Sector B	θ'	0.25
Interest rate smoothing	ρ^r	0.5
Interest rate policy response to output growth	ρ^y	0
Interest rate response to inflation in sector A	ρ_a^π	1.5
Interest rate response to inflation in sector B	ρ_b^π	1.5
Cashback rate policy response to inflation in sector A	γ_A^π	0.5
Cashback rate policy response to inflation in sector B	γ_B^π	0.5
Persistence in technology shocks in sector A	ρ_A^z	0.95
Standard deviation of technology shocks in sector A	$\sigma_{z,a}$	0.5
Persistence in technology shocks in sector B	ρ_A^z	0.95
Standard deviation of technology shocks in sector B	$\sigma_{z,b}$	0.5
Persistence in preference shocks	ρ^ζ	0.9
Standard deviation of preference shocks	σ_ζ	0.15
Transactionary value of money over private consumption	$\frac{mv}{x}$	1.531

Table 2: Parameter values

B Full set of equilibrium conditions

B.1 Household's FOC

$$
\begin{aligned}
-\frac{U_{N,t}}{U_{X,t}} &= \frac{W_t}{P_t^{HICP}} \\
\frac{U_{X,t}}{\mathbb{E}_t\{U_{X,t+1}\}} &= \beta \frac{R_t}{\mathbb{E}_t\{\pi_{t+1}^{HICP}\}} \\
\frac{C_{A,t}}{X_t} &= (1-\phi)\left[\frac{1-\Xi_{A,t}}{1-\Xi_{B,t}} q_t\right]^{-\phi} \\
\frac{C_{B,t}}{X_t} &= \phi\left[\frac{1-\Xi_{A,t}}{1-\Xi_{B,t}} q_t\right]^{1-\phi} \\
\frac{N_{A,t}^s}{N_t} &= (1-\gamma)\left(\frac{W_{A,t}}{W_t}\right)^{\rho} \\
\frac{N_{B,t}^s}{N_t} &= \gamma\left(\frac{W_{B,t}}{W_t}\right)^{\rho} \\
\frac{W_t}{P_t^{HICP}} &= \left[(1-\gamma)\left(\frac{W_{A,t}}{P_t^{HICP}}\right)^{1+\rho} + \gamma\left(\frac{W_{B,t}}{P_t^{HICP}}\right)^{1+\rho}\right]^{\frac{1}{1+\rho}} \\
U(X_t, N_t) &= \left(\frac{X_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) \\
U_{N,t} &= -N_t^{\varphi} \\
U_{X,t} &= X_t^{-\sigma}
\end{aligned}
$$

B.2 Production sector A

$$
\begin{aligned}
\Delta_{A,t} Y_{A,t} &= z_{A,t} N_{A,t}^d \\
MC_{A,t} &= \frac{(1-\tau_A)}{z_{A,t}} \frac{W_{A,t}}{P_t^{CPI}} (1-\Xi_{A,t})^{1-\phi}(1-\Xi_{B,t})^{\phi} q_t^{-\phi} \\
p_t^* &= \frac{J_{A,t}}{JJ_{A,t}} \\
J_{A,t} &= \mathcal{M}_A ms_{A,t}(1-\phi) X_t U_{X,t} MC_{A,t} + \theta\beta\mathbb{E}_t\left\{\frac{\left(\Pi_{A,t+1}^{PPI}\right)^{\eta+1}}{\Pi_{A,t+1}^{CPI}} J_{A,t+1}\right\} \\
JJ_{A,t} &= (1-\phi) X_t U_{X,t} + \theta\beta\mathbb{E}_t\left\{\frac{\left(\Pi_{A,t+1}^{PPI}\right)^{\eta}}{\Pi_{A,t+1}^{CPI}} JJ_{A,t+1}\right\}
\end{aligned}
$$

B.3 Production sector B

$$\begin{aligned}
\Delta_{B,t} Y_{B,t} &= z_{B,t} N^d_{B,t} \\
MC_{B,t} &= \frac{(1-\tau_B)}{z_{B,t}} \frac{W_{B,t}}{P_t^{CPI}} (1-\Xi_{A,t})^{1-\phi} (1-\Xi_{B,t})^{\phi} q_t^{1-\phi} \\
\tilde{p}_t &= \frac{J_{B,t}}{JJ_{B,t}} \\
J_{B,t} &= \mathcal{M}_B ms_{B,t} \phi X_t U_{X,t} MC_{B,t} + \theta' \beta' \mathbb{E}_t \left\{ \frac{\left(\Pi^{PPI}_{B,t+1}\right)^{\eta'+1}}{\Pi^{CPI}_{B,t+1}} J_{B,t+1} \right\} \\
JJ_{B,t} &= \phi X_t U_{X,t} + \theta' \beta' \mathbb{E}_t \left\{ \frac{\left(\Pi^{PPI}_{B,t+1}\right)^{\eta'}}{\Pi^{CPI}_{B,t+1}} JJ_{B,t+1} \right\}
\end{aligned}$$

B.4 Price indices, dispersion, and terms of trade

$$\begin{aligned}
1 &= \theta \left(\Pi^{PPI}_{A,t}\right)^{\eta-1} + (1-\theta)\, p_t^{*1-\eta} \\
1 &= \theta' \left(\Pi^{PPI}_{B,t}\right)^{\eta'-1} + \left(1-\theta'\right) \tilde{p}_t^{1-\eta'} \\
\Pi^{CPI}_{A,t} &= \frac{1-\Xi_{A,t}}{1-\Xi_{A,t-1}} \Pi^{PPI}_{A,t} \\
\Pi^{CPI}_{B,t} &= \frac{1-\Xi_{B,t}}{1-\Xi_{B,t-1}} \Pi^{PPI}_{B,t} \\
\Pi^{HICP}_t &= \left(\Pi^{CPI}_{A,t}\right)^{(1-\phi)} \left(\Pi^{CPI}_{B,t}\right)^{\phi} \\
\Pi^{HIPP}_t &= \left(\Pi^{PPI}_{A,t}\right)^{(1-\phi)} \left(\Pi^{PPI}_{B,t}\right)^{\phi} \\
\Delta_{A,t} &= (1-\theta)\left(p_t^*\right)^{-\eta} + \theta \Delta_{A,t-1} \left(\Pi^{CPI}_{A,t}\right)^{\eta} \\
\Delta_{B,t} &= (1-\theta')\left(\tilde{p}_t\right)^{-\eta'} + \theta' \Delta_{B,t-1} \left(\Pi^{CPI}_{B,t}\right)^{\eta'} \\
q_t &= q_{t-1} \frac{\Pi^{PPI}_{A,t}}{\Pi^{PPI}_{B,t}}
\end{aligned}$$

B.5 Market clearing conditions and aggregation

$$Y_t = \left[\frac{1-\Xi_{A,t}}{1-\Xi_{B,t}} q_t\right]^{\phi} Y_{A,t} + \left[\frac{1-\Xi_{A,t}}{1-\Xi_{B,t}} q_t\right]^{\phi-1} Y_{B,t}$$

$$\begin{aligned}
Y_{A,t} &= C_{A,t} \\
Y_{B,t} &= C_{B,t} \\
N^d_{A,t} &= N^s_{A,t} \\
N^d_{B,t} &= N^s_{B,t}
\end{aligned}$$

B.6 Monetary policy

$$R_t = \begin{cases} R_t^{policy} & \text{if no ZLB} \\ max\left(R_t^{policy}, \epsilon\right) & \text{if ZLB} \end{cases}$$

$$log\left(\frac{R_t^{policy}}{\bar{R}^{policy}}\right) = \rho^r log\left(\frac{R_{t-1}^{policy}}{\bar{R}^{policy}}\right) + \rho^y log\left(\frac{y_t}{y_{t-1}}\right) + \rho_a^\pi log\left(\frac{\Pi_{A,t}^{CPI}}{\Pi_A^{\bar{C}PI}}\right) + \rho_b^\pi log\left(\frac{\Pi_{B,t}^{CPI}}{\Pi_B^{\bar{C}PI}}\right)$$

$$\Xi_{n,t} = \begin{cases} \gamma_n^\pi log\left(\frac{\Pi_{n,t}^{PPI}}{\Pi_n^{\bar{P}PI}}\right) & \text{for} \quad n \in \{A, B\} \quad \text{if CB rule} \\ \rho_n^\xi \Xi_{n,t-1} + \varepsilon_t^{\xi,n} & \text{for} \quad n \in \{A, B\} \quad \text{if CB shock} \end{cases}$$

$$M_t^{cbdc} = M_{t-1}^{cbdc} + \Xi_{A,t} C_{A,t} + \Xi_{B,t} C_{B,t}$$

B.7 Exogenous processes

$$\begin{aligned}
&\text{TFP shock (A):} && log(z_{A,t}) = \rho_A^z log(z_{A,t-1}) + \varepsilon_t^{z,a} && \varepsilon_t^{z,a} \sim N\left(0, \sigma_{z,a}^2\right) \\
&\text{TFP shock (B):} && log(z_{B,t}) = \rho_B^z log(z_{B,t-1}) + \varepsilon_t^{z,b} && \varepsilon_t^{z,b} \sim N\left(0, \sigma_{z,b}^2\right) \\
&\text{Mark-up shock (A):} && log(ms_{A,t}) = \rho_A^\lambda log(ms_{A,t-1}) + \sigma^{\lambda,a} \varepsilon_{t-1}^{\lambda,a} + \varepsilon_t^{\lambda,a} && \varepsilon_t^{\lambda,a} \sim N\left(0, \sigma_{\lambda,a}^2\right) \\
&\text{Mark-up shock (B):} && log(ms_{B,t}) = \rho_B^\lambda log(ms_{A,t-1}) + \sigma^{\lambda,b} \varepsilon_{t-1}^{\lambda,b} + \varepsilon_t^{\lambda,b} && \varepsilon_t^{\lambda,b} \sim N\left(0, \sigma_{\lambda,b}^2\right) \\
&\text{Preference shock:} && log(\zeta_t) = \rho^\zeta log(\zeta_{t-1}) + \varepsilon_t^\zeta && \varepsilon_t^\zeta \sim N\left(0, \sigma_\zeta^2\right)
\end{aligned}$$

C Steady-state

The steady-state assumes that prices are constant $\Pi^{HICP} = \Pi^{HIPP} = \Pi_A^{CPI} = \Pi_B^{CPI} = \Pi_A^{PPI} = \Pi_B^{PPI} = 1$ and equalized between the two sectors $q = 1$, the monetary authority sets cashback rates to zero $\Xi_A = \Xi_B = 0$, and total labour is normalised to unity $n = 1$. Under these conditions and given the ratio of the transactionary value of money to private consumption expenditure $\frac{mv}{x} = 1.531$, the recursive deterministic steady-state is given by:

$$
\begin{aligned}
p^* &= 1 \\
\tilde{p} &= 1 \\
N_A^s &= (1-\gamma) N \\
N_B^s &= \gamma N \\
N_A^s &= N_A^d \\
N_B^s &= N_B^d \\
Y_A &= N_A^d \\
Y_B &= N_B^d \\
Y &= Y_A + Y_B \\
X &= Y \\
C_A &= (1-\phi) X \\
C_B &= \phi X \\
W &= X^{\sigma} N^{\nu} \\
W_A &= W \\
W_B &= W_A \\
R &= \frac{1}{\beta} \\
R^{policy} &= R \\
MC_A &= \frac{1}{\mathcal{M}_A} \\
MC_B &= \frac{1}{\mathcal{M}_B} \\
J_A &= \frac{(1-\phi) MC_A X^{1-\sigma}}{1-\theta\beta} \\
JJ_A &= \frac{(1-\phi) X^{1-\sigma}}{1-\theta\beta} \\
J_B &= \frac{\phi MC_B X^{1-\sigma}}{1-\theta'\beta} \\
JJ_B &= \frac{\phi X^{1-\sigma}}{1-\theta'\beta} \\
M^{cbdc} &= X \frac{mv}{x}
\end{aligned}
$$

D Markup shocks

As highlighted in Huang and Zheng (2005), two-sector New Keynesian models tend to deliver a less convenient for the monetary authority policy problem. Especially so if we consider only the set of implementable policy rules. In contrast to single-sector models, such setups are usually characterised by a larger number of policy trade-offs that result in a welfare loss even if policy rules are optimised. Benigno (2004) demonstrates that even a simple two-sector model where asymmetric disturbances arise solely due to shifts in sector-specific productivity and both sectors are characterised by nominal rigidities does not allow a monetary authority to eliminate all sources of welfare distortions.

Markup shocks tend to complicate the policymaker's problem even in basic one-sector New-Keynesian models by creating a trade-off between stabilising inflation and output gap (Galí, 2015). Not surprisingly, asymmetric markup shocks further complicate the policy problem of a central bank. Even if one sector is assumed to be flexible (Table 3 panel f), the monetary authority can no longer achieve the Pareto optimal outcome by means of the interest rate alone; recall that Table 1 panel f demonstrated that it was possible under asymmetric technology shocks alone.

Asymmetric markup shocks produce several additional sources of distortions. First, they affect the dispersion of markups (and, therefore, prices) between firms in a given sector, but only if prices are assumed to be sticky. Secondly, they affect dispersion in average markups between sectors, opening the terms of trade gap even if both sectors are flexible. Finally, they affect the average markup of the entire economy, producing a welfare loss even in the presence of a production subsidy on labour. Using the terminology of Mankiw and Reis (2003), asymmetric markup shocks can open the output gap, sectoral output gaps and individual firm-specific output gaps in a multi-sector sticky-price setup.

Table 3 below presents results of the analysis in Section VI under both asymmetric technology and markup shocks. It is clear that the presence of markup shocks does not affect the results of Section VI.

Panel a:			θ= 0.75	and	$\theta' = 0.75$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0.064	0	12.041	0.500	-	-	10.5953	-
IR+CBR	0.970	0	2.226	0.500	-3.482	-3.482	0.4979	6.320%
Panel b:			θ= 0.5	and	$\theta' = 0.5$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0.371	0	7.377	0.500	-	-	2.3584	-
IR+CBR	0.946	0.044	1.633	0.500	-1.071	-1.071	0.1034	1.471%
Panel c:			θ= 0.25	and	$\theta' = 0.25$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0	0	2.678	0.500	-	-	0.3841	-
IR+CBR	0.872	0.208	1.597	0.500	-0.347	-0.347	0.0432	0.228%
Panel d:			θ= 0.25	and	$\theta' = 0.5$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0	0	2.228	0	-	-	0.4814	-
IR+CBR	0.766	0	5.905	0.001	-0.345	-1.888	0.0468	0.285%
Panel e:			θ= 0.25	and	$\theta' = 0.75$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0	0	4.754	0	-	-	0.4744	-
IR+CBR	0.326	1.260	1.740	0.0149	-0.342	-3.014	0.0517	0.277%
Panel f:			θ= 0.25	and	$\theta' = 0$			
Policy	ρ^R	ρ^y	ρ^π	τ	$\gamma^{\pi a}$	$\gamma^{\pi b}$	Welfare loss	λ^c
IR	0.075	0	2.320	1	-	-	0.035	-
IR+CBR	0.075	0	2.320	1	0	0	0.035	0%

Table 3: Optimised implementable rules and associated welfare gain. A case of asymmetric technology and markup shocks.